Analysis of the transient state during the operation of a centrifugal pump in conjunction with an air vessel

Analiza stanu nieustalonego podczas współpracy pompy wirowej ze zbiornikiem powietrza

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The paper presents an analysis of transient operating conditions during the cooperation of a centrifugal pump with a compressed air vessel. The considered system consists of a pump delivering liquid from a suction reservoir into a closed tank containing an air cushion subjected to compression. Unlike conventional pumping systems operating close to a steady operating point, the analysed system is characterized by continuously changing hydraulic conditions caused by the increasing pressure of the compressed gas and the water column height. This means that the pump operates under dynamic conditions, and its operating point is not a steady state. This type of operation in a centrifugal pump–air tank system has not been analysed so far. This article presents a methodology for computing the parameters of such a system in a transient state. It provides equations describing the dynamics of pump startup, changes in the rotational speed of the rotating assembly, and the acceleration of the liquid column in the system. The influence of rotational inertia, mechanical losses, and variation of the static head caused by air compression is discussed. It was emphasized that the system's characteristics change much more slowly than those of the pump during startup, which results in a periodic excess of energy transferred from the pump to the system compared to the energy lost in the system. The study discusses the applicability of similarity laws – which have not been analyzed in the available literature for the case of dynamic processes and Suter characteristics for determining instantaneous pump operating parameters under transient conditions. Attention is drawn to the limitations associated with incomplete flow similarity during off-design operation. The analyzed system may form the basis for both conventional hydrophore installations and compressed-air energy storage systems employing pumps operating reversibly as turbines (PATs).
Keywords: centrifugal pump, air vessel, energy storage, transient state

W artykule przedstawiono analizę stanu nieustalonego podczas współpracy pompy wirowej ze zbiornikiem zawierającym sprężone powietrze. Rozważany układ obejmuje pompę tłoczącą ciecz ze zbiornika ssawnego do zamkniętego zbiornika, w którym następuje sprężanie poduszki powietrznej. W przeciwieństwie do klasycznych układów pompowych pracujących w pobliżu ustalonego punktu pracy, analizowany układ charakteryzuje się ciągłą zmianą charakterystyki hydraulicznej wynikającą ze wzrostu ciśnienia sprężanego gazu i wysokości słupa wody. Oznacza to, że pompa pracuje w warunkach dynamicznych, a jej punkt pracy nie jest stanem ustalonym. Ten rodzaj pracy w układzie pompa wirowa – zbiornik powietrza nie był dotychczas analizowany. Artykuł przedstawia metodykę obliczania parametrów takiego układu w stanie nieustalonym. Podane są równania opisujące dynamikę rozruchu pompy, zmiany prędkości obrotowej zespołu wirującego oraz przyspieszanie słupa cieczy w układzie. Omówiono wpływ momentu bezwładności, strat mechanicznych oraz zmian statycznej wysokości podnoszenia związanej ze sprężaniem powietrza. Podkreślono, że charakterystyka układu zmienia się znacznie wolniej niż charakterystyka pompy podczas rozruchu, co powoduje okresową przewagę energii oddawanej przez pompę do układu od tej traconej w układzie. W pracy omówiono możliwość wykorzystania praw podobieństwa – nie analizowanych w tym kontekście w dostępnych publikacjach oraz charakterystyk Sutera do wyznaczania chwilowych parametrów pracy pompy w stanach przejściowych. Zwrócono uwagę na ograniczenia wynikające z niepełnego podobieństwa przepływu podczas pracy poza punktem projektowym. Analizowany układ może stanowić podstawę zarówno klasycznych instalacji hydroforowych, jak i systemów magazynowania energii wykorzystujących sprężone powietrze oraz pompy pracujące rewersyjnie jako turbiny (PAT).
Słowa kluczowe: pompa odśrodkowa, zbiornik powietrza, magazynowanie energii, stan przejściowy

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Introduction

For many decades, compressed air has been one of the most widely used energy media in technology. It is used in industrial automation, pneumatic drives, energy storage, and safety systems. At the same time, pumps – and centrifugal pumps in particular – are among the basic devices used to transport liquids and build pressure in hydraulic systems. The combination of these two fields – hydraulics and pneumatics – opens up interesting possibilities in which compressed air is not merely an auxiliary medium but an active element in the process of energy storage or recovery.

In engineering applications, compressed air can interact with a pump in many ways. It can stabilize pressure in a pressure booster system by limiting the number of motor starts and compensating for temporary fluctuations in flow demand. In more advanced applications, compressed air serves as an energy storage medium: the pump forces water into an air vessel, compressing the gas inside, and the stored energy can later be used to drive the fluid back through a turbine, a pump operating in reverse, or another power recovery system.

A distinctive feature of this type of system is that the pump operates in a transient state. As the storage tank fills, the volume of the air cushion changes, and with it the back pressure acting on the pump. This results in a continuous change in the system's static head and, consequently, its overall performance characteristics. In contrast to conventional operation under relatively constant parameters, a pump working with a compressed air vessel operates under dynamic conditions, requiring the analysis of transient responses.

The dynamics of the air compression process by the pump

Regardless of whether the air vessel is a typical pressure vessel in booster pump set or a vessel for accumulating potential energy recovered in the PAT (Pump As Turbine), the centrifugal pump starts operation from a stopped state, i.e., zero rotational speed, and increases this speed until it reaches the rated value. Its performance curve rises from the starting point to the Q-H curve corresponding to rated conditions (Fig. 1).

Changes in the angular momentum of the pump's rotating assembly are given by the dynamic equation (1):

$$M_s(\omega) = I\frac{d\omega}{dt} + M_m + M_p \tag{1}$$

where: I – the total moment of inertia of all rotating masses in the pump assembly

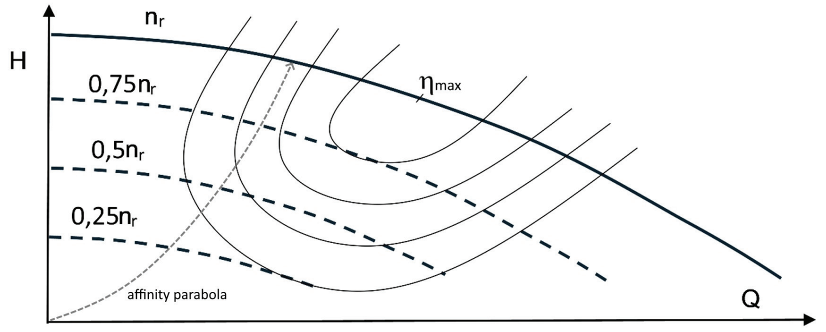

Fig. 1 Changes in pump characteristics due to an increase in rotational speed

Rys. 1. Zmiany charakterystyk pompy na skutek wzrostu prędkości obrotowej

(i.e., the pump's rotating assembly, couplings, gearbox (if any), and the electric motor's rotor), $M_s(\omega)$ – the motor's torque (as specified by the manufacturer), $M_p = P/\omega$ – the pump torque, which varies during pump operation and acts as a braking torque on the drive, $M_m = P_m/\omega$ – the sum of the friction torques occurring in the pump gland and bearings, as well as in the motor bearings and fan ω – angular velocity of the rotating assembly, $P_m = P_{mp} + P_{ms}$ – mechanical power lost due to friction in the pump and motor.

For a pump, the mechanical loss power can be determined using formula [2]:

$$\frac{P_{mp}}{P_{opt}} = 0{,}0045\left(\frac{Q_{ref}}{Q}\right)^{0{,}4}\left(\frac{n_{ref}}{n}\right)^{0{,}3} \tag{2}$$

where: Q_{ref} = 1 m^3/s, n_{ref} = 1500 1/min

Mechanical losses in induction motors typically account for about 2-10% [1] of total rated power losses; therefore, assuming P_s is the motor's shaft power and η_s is the motor's efficiency:

$$\frac{P_{ms}}{P_s\left(\eta_s^{-1} - 1\right)} = 0{,}02 \div 0{,}1 \tag{3}$$

When the engine starts, static friction is present, which then changes to kinetic friction.

Assuming the availability of the pump's efficiency area $\eta(Q,H)$ and applying the relationship for shaft power $\rho gQH/\eta$ – equation (1) can be written as:

$$M_s(\omega) = I\frac{d\omega}{dt} + \frac{P_m}{\omega} + \frac{\rho gQH}{\omega\eta} \tag{4}$$

Studies show that the time required to reach rated speed for a large pump with a motor power of 1.1 MW and a moment of inertia of rotating masses of I = 1050 kgm^2 is less than 3 seconds [3, 4]. For much smaller pumps, these times are on the order of fractions of a second (author's own research).

During operation, the pump must accelerate a mass of water $m = \rho\lambda A$ in the section of the system extending from the lower reservoir (well or suction tank) to the compressed air vessel. The initial flow rate is zero and gradually increases with an acceleration $\frac{dc}{dt} = \frac{1}{A}\frac{dQ}{dt}$, where A is the average cross-sectional area of the pipe and l is its length. The dynamics of these changes are expressed by the equation for pump head:

$$H(Q,\omega) = H_{stat} + aQ^2 + \frac{l}{gA}\frac{dQ}{dt} \tag{5}$$

H_{stat} is the static head of the pumping system, and a is the loss coefficient for that system.

Interaction between the pump and the vessel

The pumping system consists of a lower (suction) tank, piping, fittings (including a check valve), and an upper tank. A centrifugal pump delivers water from the open suction tank to the closed discharge vessel filled with air (Fig. 2). The rising water column Hw compresses the air.

The initial air pressure in the upper tank is p_1, and the final pressure, at which the

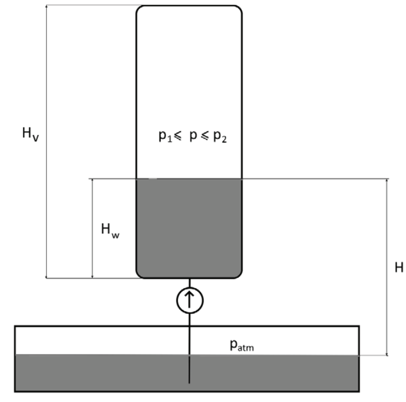

Fig. 2. Pump and lower and upper tank diagram

Rys. 2. Układ pompy i zbiorników dolnego i górnego pump is stopped by the automatic control system, is p_2.

Due to the slow process, the air compression operation is isothermal.

As compression proceeds and the water column in the air tank rises, the static part of the pumping system's head increases.

$$H_{stat} = \frac{p}{\rho g} + H_w = \frac{p_1 V_1}{\rho g A\left(H_v - H_w\right)} + H_w \quad (6)$$

Changes in the system characteristics occur much more slowly than changes in the pump characteristics due to the rapid increase in its rotational speed. This results in the pump gaining an energy advantage over the system, leading to an acceleration of the flow (Equation 5).

The pump never reaches a steady-state operating point in the system (Fig. 3). During startup, the system curve stays very low due to the system's low static head and remains flat due to low flow resistance. After the air is compressed to the required value p_2, the pump is stopped by the automatic control system. The first term in equation (6), resulting from the air pressure, can be many times greater than the second term, resulting from the water column height H_w.

Determining the head and torque

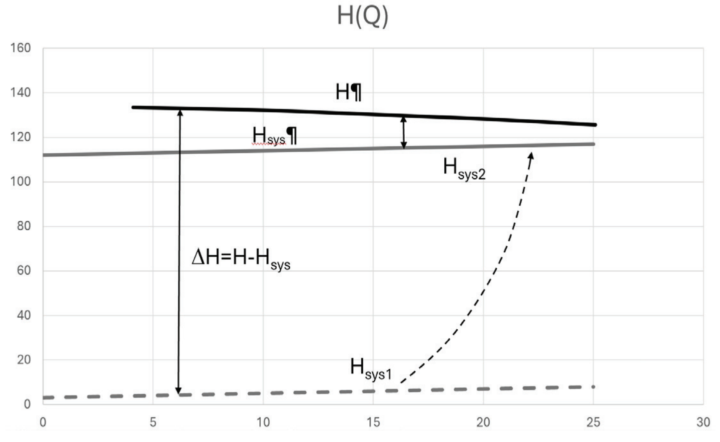

Fig. 3. Example performance curves of the pump and system

Rys. 3. Przykładowe charakterystyki pracy pompy i układu

Characteristics of a pump during air compression

To determine the pump's performance, it is necessary to know the pump head function $H(Q,\omega)$ and the pump torque $M_p(Q,\omega)$. The function $H(Q,\omega)$ can be determined using the laws of similarity. According to these laws, the head varies with the square of the rotational speed, while the flow rate varies linearly; hence, the relationship for $H(Q,\omega)$ takes the form:

$$H(Q,\omega) = \left(\frac{\omega}{\omega_r}\right)^2 f\left(\frac{Q}{(\omega/\omega_r)}\right) \quad (7)$$

For $\omega = \omega_r$ the basic pump characteristic $H = f(Q)$ is obtained.

The flow curve can often be approximated by the parabola $H = H_0 - c \cdot Q^2$. In that case, the surface $H(Q,\omega)$ takes the form:

$$H(Q,\omega) = \left(\frac{\omega}{\omega_r}\right)^2 \left[H_0 - c\left(\frac{Q}{(\omega/\omega_r)}\right)^2\right] \quad (8)$$

The torque curve of a pump M_p is not typically measured by the manufacturer; instead, the shaft power curve is provided.

Given that the pump's power is proportional to the cubic of the rotational speed, and thus the torque is proportional to the square of the rotational speed, it can be determined, similarly to the head, using the following equation:

$$M_p(Q,\omega) = \left(\frac{\omega}{\omega_r}\right)^2 g\left(\frac{Q}{(\omega/\omega_r)}\right) \quad (9)$$

As with the head, for $\omega = \omega_r$, the basic characteristics of the pump's torque $M_p = g(Q)$ or power $P_p = \omega g(Q)$ are obtained.

Similarity laws are based on the assumption of similarity of velocity fields at different rotational speeds. However, this assumption is not accurate, because the sizes of separation zones and vortices formed when the pump operates outside the design point do not scale. Similarly, the thicknesses of boundary layers do not satisfy the similarity laws.

Sometimes, the efficiency $\eta(Q,H)$ measured by the manufacturer is available. To compute the dynamics of the system during air compression by the pump, its analytical form is required – which is difficult to obtain. There are conversion formulas for maximum efficiency as a function of rotational speed, e.g., [2], but since they apply to only a single point, they cannot be used.

Another method is to use Suter's characteristics [8], which present the full operating range of the pump in the form of dimensionless coefficients of head WH and torque WM, representing the double-reduced head and torque.

$$\left.\begin{aligned} WH(X) &= \frac{h}{\alpha^2 + q^2} \\ WM(X) &= \frac{h}{\alpha^2 + q^2} \end{aligned}\right\} \quad (10)$$

where:

$$X = i \cdot \pi + arctg\frac{q}{\alpha}, \quad i = \begin{cases} 0 \text{ for } 0 \le X \le \pi \\ 1 \text{ for } \pi \le X \le \frac{3}{2}\pi \\ 2 \text{ for } \frac{3}{2}\pi \le X \le 2\pi \end{cases} \quad (11)$$

$$h = \frac{H}{H_n}, \quad q = \frac{Q}{Q_n}, \quad m = \frac{M}{M_n}, \quad \alpha = \frac{\omega}{\omega_n}$$

A full description of the approximation of these coefficients is provided, among other places, in [5].

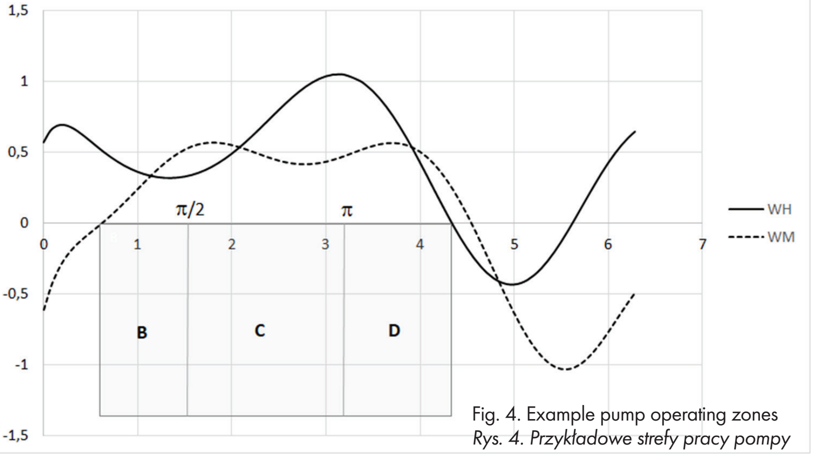

Fig. 4. Example pump operating zones

Rys. 4. Przykładowe strefy pracy pompy

Figure 4 shows example characteristics of *WH* and *WM* as a function of the dimensionless parameter X. Region D represents normal pump operation during air compression, when $H > 0$, $Q \geq 0$, $n \geq 0$, and $M > 0$ for $\pi \leq X \leq 1.38\pi$.

WH and *WM* characteristics are available for selected specific speeds [3,4,5]. If the pump being analysed has a different specific speed, the WH and WM should be interpolated, for example, linearly, based on the available measured values values.

Publication [6] presents an analysis of changes in the hydraulic parameters of a nuclear power plant system resulting from a failure, using Suter characteristics.

Conclusions

The method presented in this article allows for the calculation of the parameters of a pump-air vessel system during the compression of air by pumping in water. This process is transient in nature due to the changing pump characteristics resulting from an increase in rotational speed, as well as the changing system characteristics resulting from an increase in air pressure and the water column height in the tank.

During operation, the pump's performance characteristics change much more rapidly than those of the pumping system, which leads to the pump's parameters exceeding those of the system and to an increase in the mass of the pumped water. The pump and the system never reach a state of equilibrium.

To determine the pump head $H(Q,\omega)$ and pump torque $M_p(Q,\omega)$ within the range of its positive parameters (the so-called first quadrant), similarity laws can be used, which allow the determination of the pump's instantaneous operating point. An alternative is to use the Suter characteristic.

The accuracy of a model based on similarity laws decreases during operation far from the design point, when separations and non-scalable vortex structures occur. Additionally, the thicknesses of the boundary layers are not subject to similarity. Despite these limitations, they are commonly used to determine pump parameters as a function of velocity [6].

The analysed system can be applied not only in classic pressure booster systems but also in compressed air energy storage systems and energy recovery systems with a pump operating in reverse as a turbine.

Further work should include a dynamic and energy analysis of the entire compression and energy recovery process in pump-as-turbine (PAT) mode.

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